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Testing cosmological models with COBE data (*)

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Summary. — We test cosmological models with $\Omega < 1$ using the COBE two-year cross-correlation function by means of a maximum-likelihood test with Monte Carlo realizations of several Ω models. Assuming a Harrison-Zel′dovich primordial power spectrum with amplitude $\propto Q$, it is found that there is a large region in the (Ω, Q), parameter space that fits the data equally well. We find that the flatness of the universe is not implied by the data. A summary of other analyses of COBE data to constrain the shape of the primordial spectrum is presented.

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1. – Introduction

Since the discovery of anisotropies [1, 2] the cosmic microwave background (CMB) has become a powerful tool to probe the prevailing conditions in the early universe (see the review by White *et al.* [3]). Structure formation scenarios, in particular, can receive important restrictions based on the measured $\Delta T/T$. According to the standard model, the formation of large-scale structure in the universe has its origin in the growth of primordial inhomogeneities in the matter distribution. The power spectrum, $P(k) \propto \langle \delta_k^2 \rangle$, gives the variance of each Fourier mode. CMB photons climbing out of gravitational potential wells caused by overdensities on the surface of last scattering suffer red-shifts (the Sachs-Wolfe effect [4]).

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2. – The shape of the primordial spectrum

The final product of an ideal (noiseless) CMB experiment is one realization on the 2D sphere of the random fluctuating temperature field. If the fluctuations are Gaussian, the field is completely specified by the angular power spectrum [5], C_l, defined in terms of an expansion of the CMB temperature autocorrelation function $C(\alpha)$ in Legendre polynomials:

$$\left\langle \frac{\Delta T(\hat{n}_1)}{T} \frac{\Delta T(\hat{n}_2)}{T} \right\rangle = \frac{1}{4\pi}(2l+1)\sum_l C_l P_l(\cos\alpha), \tag{1}$$

where α is the angle separating two directions in the sky, $\cos\alpha = \hat{n}_1 \cdot \hat{n}_2$.

The temperature variations on the surface of the sphere can also be expressed as an expansion in spherical harmonics:

$$\frac{\Delta T}{T} = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi), \tag{2}$$

where the dipole term, $l = 1$, has been left out as this is dominated by the local Doppler effect induced by the motion of the observer relative to the CMB. The angular power spectrum in terms of the harmonic coefficients is $C_l = \langle |a_{lm}|^2 \rangle$.

Knowledge of the spectrum of primordial density fluctuations allows one to compute the r.m.s. mass fluctuation on a given mass scale $(\delta M/M)_R$ and the peculiar velocity field. The need for a precise determination of $P(k)$ is well established. $P(k)$ is usually parametrized as a power law $P(k) = Ak^n$, with n the spectral index and A a normalization constant proportional to the r.m.s. quadrupole moment, Q. For small k and $\Omega = 1$ it is possible to find an analytic solution [5] to the Sachs-Wolfe integral that connects the CMB angular power spectrum with the parameters n and Q.

There have been several analyses of the COBE data (see table I) using different techniques to place constraints on n and Q (assuming $\Omega = 1$); however, an examination of these results reveals several points worth discussing: 1) The numbers obtained are somewhat dependent on the type of analysis performed and the data preparation strategy (for example, whether the quadrupole is included or not). A clear convergence is not obvious. 2) The inflation-motivated spectrum $n = 1$ is generally consistent with the data and only in two cases it is marginally consistent. 3) With the proliferation of cosmological model parameters, large angular scale $\Delta T/T$ data alone is not enough to pin down the best model. 4) The error bars are approaching the limit of minimum uncertainty allowed by cosmic variance within the available range in l space. These facts underline the need to perform experiments such as COBRAS/SAMBA which extend the experimental window to higher l's.

The first point is naturally expected as a result of the intrinsic limitations and the low signal-to-noise ratio available at the moment. It is clear that quadrupole removal has some influence on the results. The second point may in fact be underlining the need to test other cosmological parameters (*i.e.* $\Omega < 1$, tilted spectra, non-vanishing cosmological constant, etc). The range of n values consistent with the data goes from 0.72 to 1.46, and when the 1-σ error bars are taken into account, this interval extends to 0.59–1.85. This is clearly not a strong constraint on the spectral index; even though the Harrison-Zel′dovich spectrum receives support there is room for other theoretical

TABLE I. – *Values for the parameters that define the spectrum of primordial density fluctuations, Q and n.* COBE1, COBE2 *are* COBE's *first- and second-year sky maps, respectively.* ξ_{gg} *is the two-point galaxy-galaxy correlation function,* ξ_{v} *is the velocity correlation function. The last two entries of the table are given for the special case* $n = 1$, *but the original results are given as expressions that make explicit the* Q, n *degeneracy:* $Q(n) = 22.2 \pm 1.7 - (4.7 \pm 1.3) \times n$, *and* $Q(n) = (15 \pm 2.6) \exp[0.46(1-n)]$, *respectively.*

Method	Data	n	Q (μK)	Ref.
Correlation function	COBE1 and ξ_{gg}	0.8 ± 0.3		[6]
Genus	COBE1	1.2 ± 0.3		[7]
Correlation function	COBE1, ξ_{gg}, ξ_{v}	0.95		[8]
Correlation function	COBE1	1.1 ± 0.5		[1]
σ_8	COBE1	1.0 ± 0.23		[9]
σ_{sky}	COBE1	0.72 ± 0.13		[10]
Correlation function	COBE1	1 fixed	14.5 ± 1.7	[11]
Peak analysis	COBE1	1 fixed	18 ± 5	[12]
Peak analysis	COBE2	1 fixed	17 ± 3	[13]
S/N Eigenmode	COBE2	1.4 ± 0.4		[14]
Cross-power spectrum	COBE2	1 fixed	19.8 ± 2.0	[15]
Cross-power spectrum	COBE2 $l \leqslant 30$	$1.25^{+0.4}_{-0.45}$		[15]
Cross-power spectrum	COBE2 $l \leqslant 19$	$1.46^{+0.39}_{-0.44}$		[15]
Orthonormal functions	COBE2	1.22	$17^{+7.6}_{-4.8}$	[16]
Orthonormal functions	COBE2 (no quad)	1.02	$20^{+10.5}_{-6.5}$	[16]
Orthonormal functions	COBE2	1.10 ± 0.32		[16]
Orthonormal functions	COBE2 (no quad)	0.87 ± 0.36		[16]
Orthonormal functions	COBE2	1 fixed	19.9 ± 1.6	[16]
Orthonormal functions	COBE2 (no quad)	1 fixed	20.4 ± 1.7	[16]
Temperature r.m.s.	COBE2	1 fixed	$17.0^{+2.5}_{-2.1}$	[17]
Temperature r.m.s.	COBE2 (no quad)	1 fixed	$19.4^{+2.3}_{-2.1}$	[17]
Correlation function	COBE2	$1.42^{+0.49}_{-0.55}$	$14.3^{+5.2}_{-3.3}$	[2]
Correlation function	COBE2	1 fixed	18.2 ± 1.5	[2]
Correlation function	COBE2	1.42 ± 0.37		[2]
Correlation function	COBE2 (no quad)	$1.11^{+0.6}_{-0.55}$	$17.4^{+7.5}_{-5.2}$	[2]
Correlation function	COBE2 (no quad)	1 fixed	18.6 ± 1.6	[2]
Correlation function	COBE2 (no quad)	1.11 ± 0.4		[2]
Pixel temperature	COBE2	1.11 ± 0.29	20.2 ± 4.6	[18]
Correlation function	TENERIFE	1 fixed	26 ± 6	[19]
Cross-correlation	COBE + TENERIFE	> 0.87		[19]
Genus	COBE1	1 fixed	17.5 ± 2	[20]
Correlation function	COBE1	1 fixed	15 ± 2.6	[21]

possibilities. A spectral index $n < 1$ may arise in a universe with a strong gravitational-wave background [22].

3. – CMB anisotropies in open cosmological models

Motivated by the previous discussion we have performed a statistical test for models with $\Omega < 1$. The evidence in favour of an open universe is overwhelming [23]. Open universes have not been studied in detail because the precise shape of the density power spectrum beyond the curvature scale in these universes has not been definitely determined. Here we study the CMB angular correlation with the simplest form of $P(k)$ (*i.e.* the scale-invariant, $n = 1$ spectrum) using the harmonic coefficients for open models given by Sugiyama [24].

The method consists in a maximum-likelihood test [25, 26] in which the COBE two-year cross-correlation function, $C(\alpha)$ (53×90), is compared with several Monte Carlo realizations of cross-correlation functions from models with varying Ω and Q. The Monte Carlo realizations of $C(\alpha)$ functions were generated by the following procedure: first, a set of random harmonic coefficients with variance given by the particular model is generated (up to $l = 30$); from these coefficients two sky maps are produced following the same pixelization scheme as in the COBE-DMR data set and taking into account the beam shape; instrumental noise with a level consistent with the sensitivities for each channel is added to the maps; and finally, a galaxy cut ($|b| < 20^\circ$) is done and the cross-correlation computed.

A grid of Monte Carlo data sets were generated for Ω values: 0.1, 0.2, 0.3, 0.4, 0.6, 0.8 and 1.0; and Q in the range: 7–30 μK in steps of 2 μK. With these Monte Carlo data we looked for the models that maximized the likelihood. It is found that there exists a degeneracy in the plane (Ω, Q) that maximizes the likelihood value. To obtain the $\Omega = \Omega(Q)$ relation we first assign the standard deviation of the data as error to each value of Q (for fixed Ω) for which the likelihood is a maximum; then a fit of the (Ω, Q) points is done to a cubic polynomial, which results in $Q(\Omega) = 10.67 + 55.81\Omega - 128.59\Omega^2 + 81.26\Omega^3 \mu\mathrm{K}$. A qualitative origin of such a relation can be understood in terms of the combined contributions of potential fluctuations and the integrated effect due to curvature which results in partial cancellation near $\Omega \approx 0.4$. The model that gives the maximum likelihood is $\Omega = 0.1$, $Q = 15$ μK, however; all other models along the $Q(\Omega)$ cubic polynomial do not show a significantly worse fit. Thus, COBE-DMR data do not favour a flat universe with $\Omega = 1$.

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