Eratosthenes visits middle school: Assessing the ability of students to work with models of the Earth

by Sergio Torres and Judith L. Powers

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Students in Bethesda measuring the shadow

Imagine the excitement of middle school students who participate in a scientific project with students in a remote country, and come home to tell their parents, "Mom, Dad, I measured the Earth today!" That is precisely what we did in our science class, using meter sticks and measuring their shadows in two distant locations. To obtain the size of the Earth, students had to understand the connection between the measurements of the shadows and a model of the spherical Earth following the method developed by Eratosthenes. In the process, students learned about the history of science and the value of collaborating with students in an international setting, and appreciated the fact that a complex and large object such as our planet can be measured using simple geometric concepts.

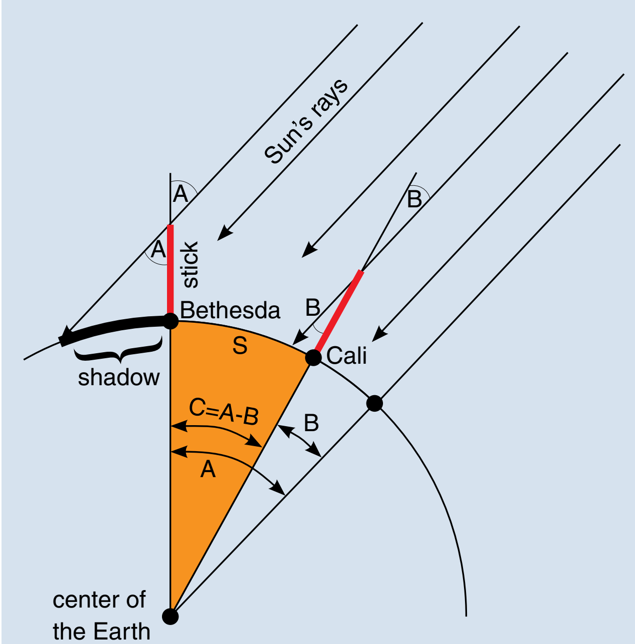
Background

The method of Eratosthenes (circa 273–194 BCE) to estimate the Earth's girth consists of measuring the distance separating two distant locations along the north-south line together with their angular separation, determined by measuring the shadows cast by sticks at the two locations (Sarton 1952; Ferguson 1999). Eratosthenes was the director of the Great Library of Alexandria, the greatest library of antiquity. From this enviable position he had access to the most current knowledge in geography, math, and every scientific project of importance. Born in the Greek colony of Cyrene in northern Africa, he received the best education available at the time and became a well-respected scholar in a wide variety of fields, including math and geography. He developed a method to make maps using lines of reference (the precursors of latitude and longitude) and assigning coordinates to geographic places.

To illustrate Eratosthenes' observations, we made a preliminary demonstration in class by shining light on models of a flat and spherical Earth with two sticks of the same size inserted perpendicularly to the surface of each. Whereas the shadows projected on a flat Earth are equal, those on the sphere have different lengths, depending on their location. For the round Earth model, we used an inexpensive ($9) Styrofoam ball of 25 cm (10 inches) in diameter. Similarly, a flat slab of Styrofoam can easily become a model for a flat Earth. These materials are readily found at a hardware store and should present no safety issues. The ensuing discussion with students resulted in the realization that replicating Eratosthenes' method would be an exciting hands-on activity that would offer multiple opportunities to involve students in the scientific process. To start the discussion, we asked students to think about how they can show that the Earth is round. One student mentioned the evidence furnished by observation of a ship as it navigates away from the shore (first the hull disappears beneath the horizon, then the sails, and finally the top of the sails). An effective method to determine if students understand the effect of the roundness of the Earth in this case is to ask them to illustrate it with their own drawings.

Measuring the size of the Earth is a suitable way to begin the development of a precise model of the Earth and the solar system, consistent with the recommendations of the National Science Education Standards (Agan and Sneider 2004). The Earth is modeled as a sphere; to connect it with the measurements, we used a circle representing the cross section of the Earth that passes through the two measurement locations. The geometry, as depicted in Figure 1, is defined by this circle and a wedge-shaped section (or slice) as shown. The number of slices in the circle is 360° divided by the angle of the slice (C), and the circumference of the Earth is the distance (S) multiplied by the number of slices (Webb and Bustin 1988):

FIGURE 1 Geometry that relates the measurements of shadows with the angle of the slice



Earth's circumference = S x (360°/C)

Replicating Eratosthenes's measurements

For this activity, students from Holton-Arms School in Bethesda, Maryland, teamed with students of Liceo Benalcazar in Cali, Colombia. The latter, located 3,936 km directly south of Bethesda, was chosen because it meets the conditions for Eratosthenes's method, namely that the places are located in the same meridian and distant enough so that the curvature effects are manifested. Teachers interested in locating a partner group to do the experiment should look at cities located directly south or north of their home city at a distance of 200 km or larger. Larger distances result in smaller errors.

We established a project website (Torres 2008) with reference material to help teachers in both locations prepare for the activity. Pictures and results were added later.

Measuring the shadows

To determine the angle of the slice (C), we needed the angles of the meter sticks with the Sun's rays mea-

FIGURE 2 Shadow measured in the Cali location

sured simultaneously in the two cities when the Sun reached its highest altitude. Measuring the shadows of the sticks allowed us to compute these angles.

We arranged to take simultaneous measurements on March 11, 2008, at 13:18 (daylight saving) in Bethesda and 12:18 in Cali, the time the Sun reaches its highest altitude (USNO). The particular date chosen for the measurement has no significance; however it is of historic interest to mention that Eratosthenes made the measurements on the day of summer solstice because on that day the Sun is directly overhead in one of the places he chose. Because there is no shadow cast when the Sun is directly overhead, his measurements were made easier. In fact, on the island of Elephantine in the Nile there remains a deep well that Eratosthenes used to determine that the Sun at noon was directly overhead (it does not project shadows on the walls of the well). In Bethesda, we distributed meter sticks and meter tapes to 13 groups of approximately five students each (a similar arrangement was done in Cali). Because the activity has to take place at the right time, the activity was coordinated with teachers from other classes. In Bethesda, we had three classes. Because it was cloudy, we took measurements starting a few minutes prior to the target time and continued taking them through 13:19. We had students check that their meter sticks were perpendicular to the ground by simple observation (a carpenter's level could be used for more precision).

Pictures of the shadows were taken during the measurement and shared on the project website. The most dramatic effect, which was readily appreciated by students, was the visible difference in the length of the shadows (92 cm in Bethesda versus 13 cm in Cali) when viewing the pictures side by side (see opening photo and Figure 2).

Calculating the circumference of the Earth in class

Using an LCD projector, we showed students how we could estimate the circumference of the Earth by measuring the angle of the slice. We divided the task into three steps: (1) determining the angle of the slice, (2) estimating the Bethesda-Cali distance, and (3) computing the Earth's circumference.

Measuring the angular separation of the two cities

We instructed students to draw a right triangle with the measured lengths of the stick and the shadow scaled down by a factor of 10 to make it fit on a sheet of paper. The triangle was built with the shadow as the base and the stick as the perpendicular side. Students measured the angle opposite the base with a protractor. This was done for the observations from the two cities (angles A and B). The angle of the slice is the difference: C = A – B. It would be beneficial to involve a math/geometry teacher in this part of the project to ensure that students understand the geometry behind Eratosthenes's method.

The challenge was to find a method to estimate the distance between the two locations that did not require prior knowledge of the radius of the Earth. Eratosthenes sent surveyors to walk the entire distance. We thought that parents would be reluctant to subject their children to a 1,357 km walk, combined with 2,579 km of swimming; thus, we settled for the more practical solution of measuring the distance on flat maps. After all, it was Eratosthenes himself who made the first maps by locating points on a longitude-latitude grid.

We used large maps from a National Geographic atlas. To put maps on a flat page, cartographers use various projection methods, and it is well known that maps that include large extensions (such as the Cali–Bethesda distance) invariably result in distortions of those regions of the map further away from the center of the map. To reduce errors due to these projection effects, we assembled a mosaic of three maps covering the Bethesda-Cali great circle. Students measured distances in centimeters directly on the maps, and from the kilometers scales on the maps they calculated how many centimeters corresponded to 1 km.

Managing the measurements from three maps with different scales and making sure that the measurements covered a continuous arc were the main sources of errors. We observed that, for most students, this step required spatial reasoning skills that not all had acquired. After the exercise was done, we realized that it would have helped students if they had a chance to practice the distance measurement on a map before the actual measurement. Students in the first group were more concrete learners with many questions about the individual steps and how they related to our final solution. The process was smoother with the second class, because we knew where questions would arise and students in this group were more analytical.

We used an LCD projector to show the angles that we measured, and then used the diagram to reiterate why we were following the steps in the experiment. Once the experiment was completed and reviewed, students were visibly pleased that they had replicated Eratosthenes's method for determining the circumference of the Earth. To assess students, teachers can ask them to explain why each step of the project was necessary, what specific information was obtained at each step, and how they put all the information together.

Results

We analyzed the report sheets to assess the accuracy of the results. For the angle C, the average of all the measurements came to within 2° of the actual value of 35.5° (USNO). The average of the distance S was 3,722 km. The actual distance is 3,936 km, so the absolute error was 214 km. The average for the circumference was 40,231 km, for an error of 223 km (using 40,008 km for the Earth's circumference as derived from the average radius). The average came close to the actual value because of the partial cancellation of errors in C and S. Based on the spread of individual measurements, the overall error of the circumference measurement was 7%.

Lessons learned

Analyzing the data with students in class was important because we were able to identify issues and give them immediate verbal and visual explanations. We knew that the spatial reasoning skills required to use three different maps to compute the distance and to relate this computation to the circumference of the Earth would be challenging for many students. We would recommend using one map instead of a combination of maps (at the expense of a larger error due to map projection effects).

The use of an LCD projector was essential because we could show illustrations of the concepts and visually reinforce how and why we made the Sun's shadow measurements. This repetition and reiteration was critical to ensuring that students understood the entire procedure.

Working with students and teachers from another country provided a concrete illustration of how students from different backgrounds, living in different countries, can collaborate to solve scientific problems. We would have preferred more interaction between the Bethesda and Cali students. In the future, we could use internet connections to see each other's measurements in real time and to jointly discuss the analysis and results. Because the actual computation of the Earth's circumference involves the measurements obtained from both cities (i.e., the angle C is the difference of the angles A and B measured in each city), students in Cali obtained the same results for the circumference of the Earth.

Potential extensions of the project

The following extensions could be added for more advanced students: (1) The source of the seasons is the 23.5° inclination of the Earth's equator, which can be measured following the stick and shadow method during the solstices; (2) the difference between the magnetic and geographic north can be made explicit by comparing the direction of the compass needle with the direction of the shadow when the Sun is at the maximum elevation; (3) errors can be appreciated by checking how the circumference varies as we use values for C and S, adding and subtracting one standard deviation; (4) demonstrating that the Earth is not a perfect sphere by noticing that even if we make no errors in our measurements, we could have accuracy problems.

Conclusions

With actual measurements in the field, followed by analysis in the classroom, students experienced first-hand the nuances of scientific research. The geometric model was an effective tool to connect measurement with theory; adapting the geometry used by the Greeks in a tangible experiment expanded students' understanding of how mathematics is an essential component of science. They realized that even the most careful measurements are subject to errors. They were able to appreciate that length scales tremendously larger than the human scale can be measured by means within their reach. Judging by the level of engagement of students and the feedback of students and parents, we concluded that this activity increased students' interest in science and their understanding of the scientific process.

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