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COSMOLOGY AFTER COBE *

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Abstract. The measured anisotropies in the temperature of the cosmic microwave background radiation (CMB) by the Cosmic Background Explorer (COBE) are consistent with models of gravitational collapse for the formation of large scale structure in the universe. The amplitude of cosmological fluctuations on the largest scales is fixed by COBE. From COBE's data it is also possible to test for the shape of the primordial spectrum. Statistical tests using COBE's two year data and based on the geometric characteristics of anisotropy spots taking into account 'cosmic variance' and the relevant experimental details indicate that the primordial spectrum has a slope in the range $n = 0.8 - 1.3$. Possible identification of hot and cold spots of cosmological origin is also given.

1 Introduccion

Studying the posibility that primordial Helium might have been sinthesized in an early hot phase of the universe, Gamow (1948), and Hermman and Alpher(1948) realized that the radiation energy present at those early epochs must exist today but at a much lower temperature due to the expansion. This is the cosmic microwave background radiation (CMB) which was discovered in 1965 by Penzias and Wilson (1965). Since its discovery, the study of the CMB has become an important element in cosmology. The Cosmic Background Explorer (COBE) has shown that the CMB is a radiation field with the characteristics of a blackbody at $T_0 = 2.726 \pm 0.010$ K (Mather *et al.*, 1994). CMB photons last scattered at $z \approx 1000$ (also refered to as 'decoupling') when matter and radiation were in a state of thermal equilibrium at a temperature of $T \approx 1000T_0$. At this epoch fluctuations in the gravitational potential must have induced fluctuations both in the matter density and radiation density. Matter density fluctuations grew after decoupling giving origin to the large scale structure observed today. The spectrum of primordial density fluctuations (PDFs) is usually represented by a power law spectrum $P(k) \equiv |\delta_k|^2 \propto Q^2k^n$, with δ_k the Fourier component of $|\delta\rho/\rho|$ at wave number k. The spectrum of PDFs is parametrized by the spectral index n and the amplitude Q (or Q_{rms-PS} as used in the literature).

Fluctuations in the radiation field arise as a natural consequence of matter density fluctuations at the surface of last scattering which leave and imprint on the CMB via the Sachs-Wolfe effect (Sachs and Wolfe, 1967). Two satellite experiments claim detection of large angle structure on the CMB: the Relik-I mission (Strukov *et al.*, 1992a; Strukov *et el.*, 1992b; Sazhin *et al.*, 1993) and COBE (Smoot *et al.*, 92).

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2 COBE Overview

COBE is NASA's first space mission entirely devoted to cosmology. The 2.27 ton satellite was launched on 18 November 1989 from Vandenberg Air Force Base in California. In November of 1993 it completed its four year mission and its instruments were switched off. COBE carried three experiments on board: the Far-Infrared Absolute Spectrophotometer (FIRAS) to measure the CMB spectrum, the Differential Microwave Radiometers (DMR) to measure temperature anisotropies in the microwave background, and the Diffuse Infrared Background Experiment (DIRBE) to measure the infrared background. In this paper I will only deal with the DMR project and its scientific results. For a status and overview of the COBE project see Bogges *et al.* (1992), Bennett *et al.* (1993a), Mather *et al.* (1993) and Gulkis *et al.* (1990).

Thus far, the only existing full sky maps of the CMB taken from space are those of RELIKT and DMR-COBE. The latter is a multi-frequency experiment (31.5, 53 and 90 GHz), thus allowing for the subtraction of the diffuse emission from our galaxy. The DMR instruments are described in detail by Smoot *et al.* (1990), and consist of two pairs of radiometers at 90 and 53 GHZ and one radiometer at 31.5 GHz. Each radiometer has two connical antennae of 7° FWHM beam-width pointing at directions separated by 60° (the two 31 GHz pairs of antennas share the same radiometer). The receiver electronics forms the temperature difference between these directions, and the end result of the data analysis is a total of 6 sky maps, one for each channel.

Preliminary results of the first 6 months of observation by DMR placed an upper limit on CMB fluctuations $\Delta T/T < 4 \times 10^{-5}$ (95% C.L.) (Smoot *et al.* 1991). The detection of anisotropies was reported based on the analysis of the first year data. These results are: a) the measurement of the rms sky temperature on a 10° scale, $\sigma_{sky}(10^\circ) = 30 \pm 5\,\mu$K; b) the CMB temperature correlation function, c) the amplitude and direction of the dipole (see below) and quadrupole ($Q_{rms} = 13 \pm 4\,\mu$K) and d) the inferred parameters of the power spectrum of PDFs: $Q = 16 \pm 4\,\mu$K, $n = 1.1 \pm 0.5$ (Smoot *et al.*, 1992). Separate papers present the analysis of systematic errors (Kogut *et al.*, 1992), the modeling of the galactic contribution (Bennett *et al.*, 1992a), the calibration of the instrument (Bennett *et al.*, 1992b), and the interpretation of the measurements (Wright *et al.* 1992). A detailed analysis of the statistic moments and the topological caracteristics of COBE maps has been used to test consistency with Gaussian PDFs (Smoot *et al.*, 1994). The three point correlation function of the first year data is also consistent with Gaussian fluctuations (Hinshaw *et al.*, 1993). The results of the analysis of the second year data set have been reported in Bennett *et al.* (Bennett *et al.*, 1994) and Kogut *et al.* (1994) and are consistent with the first year results but somewhat smaller error bars: $\sigma_{sky}(10^\circ) = 30.5 \pm 2.7\,\mu$K, $Q_{rms} = 6 \pm 3\,\mu$K, $Q = 12.4^{+5.2}_{-3.3}\,\mu$K, and $n = 1.59^{+0.49}_{-0.55}$. Wright *et al.* (1994), and Górski *et al.* (1994) deal with the computation of the CMB angular power spectrum. The signal detected by COBE-

DMR has been independently confirmed by the ballon FIRS experiment (Ganga *et al.*, 1994), and the ground based TENERIFE experiment (Hancock *et al.*, 1994).

3 Features of the Microwave Sky

3.1 THE DIPOLE AND QUADRUPOLE

The most important contributions to the signal detected by DMR are the continuum galactic emission and the dipole anisotropy. The galactic signal in the frequency range important for DMR is mainly due to the radio emission of relativistic electrons moving in the galactic magnetic field, and thermal emission of ionized hydrogen clouds. For the type of analysis in which we are interested the galactic signal becomes the largest unwanted systematic effect and has to be modeled and subtracted out (Bennett *et al.*,1992a) or just eliminated by masking out the galactic band prior to any analysis. Cutting a section of the sphere, however, complicates the reconstruction of the angular power spectrum because the spherical harmonics are not orthonormal anymore on a cut sphere (Wright *et al.*, 1994; Górski *et al*, 1994). The dipole anisotropy is the induced Doppler frequency shift due to our velocity relative to the CMB of 370 Km s^{-1} and has an amplitude of 3.363 ± 0.024 mK in the direction $\ell = 264.4° \pm 0.2°$, $b = 48.1° \pm 0.4°$ as measured by DMR (Bennett *et al.*, 1994; Kogut *et al.*, 1993), ℓ, b, are galactic longitude and latitude respectively.

Expresing the CMB temperature anisotropy in terms of an spherical harmonics expansion, $\Delta T/T = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$, the dipole components are given by the 3 $l = 1$ coefficients. Of second order in l comes the 5 quadrupole components ($l = 2$), which can be produced by a second order Doppler shift ($\Delta T = T_0(\beta^2/2)\cos(2\theta) = 1.2\,\mu$K), global rotation (Collins and Hawking, 1973; Barrow *et al.*, 1985), cosmic strings (Vilenkin, 1985), long wave gravitational wave (Burke, 1975; Lucchin *et al.* 1992), and PDFs. If the later effect is the dominant contribution to the quadrupole, then it has a large intrinsic variance (see below). COBE reported a quadrupole detection of $16 \pm 4\,\mu$K in the first year data and only $6 \pm 3\,\mu$K in the second year data. Due to the large cosmic variance attached to the $l = 2$ coefficients, the statistical significance of the detection is low (Gould, 1993). In addition to the limitations imposed by cosmic variance one most add the difficulties introduced by the presense of noise and the aliasing of higher harmonics with the quadrupole in a cut sphere (Stark, 1993). The galactic quadrupole is also a large systematic error in the determination of Q_{rms} (Torres, 1994c). Noise alone can generate a Q as high as $26.2\,\mu$K for a galactic cut $|b| > 20°$, and cosmic variance and non-uniform sky coverage combined with a galactic cut $|b| > 10°$ introduces a bias in Q_{rms} of $31.7\,\mu$K (Tenorio *et al.*, 1993).

3.2 HOT AND COLD SPOTS

The first year COBE-DMR skymaps have a signal-to-noise ratio close to one, while for the second year data it is slightly larger than one, thus what appears as a hot spot is not necessarily a legitimate hot spot (i.e. one of cosmological origin or one that can be attributed to a local source).

After the detection of large angle scale structure in the CMB temperature distribution by three independent experiments, identifying and characterizing hot spots becomes now more relevant, and experiments (such as TENERIFE) can design survey strategies so as to look for a particular hot spot. Knowing the geometry of hot spots may also be of great potential for testing the 'mixing of geodesics' effect (Gurzadyan and Kocharyan, 1993) which predicts the presense of highly elongated hot spots if the geometry of the universe is characterized by negative curvature. The size of anisotropy spots gives important information on the state of polarization of the CMB signal (Sazhin, 1994). A search for unresolved sources on the 2 year DMR data ($|b > 30°|$) finds no sources brighter than $192\,\mu$K at 53 GHz (Kogut *et al.*, 1994).

An ideal noise-less experiment should see hot spots of cosmic origin at all angular scales. It has been suggested that even noise limited experiments may observe the hottest spots (Sazhin, 1985). The expected number density of spots and their geometric characteristics have been derived for 2D homogeneous Gaussian random fields (Adler, 1981; Bond and Efstathiu, 1987; Vittorio and Juszkiewiks, 1987; Coles and Barrow, 1987; Coles, 1988a; Coles, 1988b; Gott *et al.*, 1990; Martìnez-Gonzàlez and Sanz 1989).

The excursion set of a random temperature field is the domain of all points in which the field takes on values $T \geq T_\nu = \nu\sigma$, where σ and ν are the field standard deviation and threshold, respectively. For high threshold levels ($\nu > 1$), the excursion set is characterized by non-connected regions (hot spots) whose shape approaches a circle as the threshold increases. Similarly, a cold spot can be defined as the region where the temperature takes on values $T \leq -T_\nu$. A negative σ is used to indicate these cold spots.

Data from the first year 53 and 90 GHz DMR radiometers, the two with the best sensitivity, were used in this analisis. The maps were scaled to thermodynamic temperature, the dipole was fitted and removed, the $1.2\,\mu$K kinematic quadrupole was removed, and finally the maps were Gaussian smoothed with a smoothing angle equal to the beam size ($\theta_s = 2.9°$) in order to reduce noise. The contribution of the galaxy is taken into account by excluding the equatorial band $|b| \leq 20°$, where b is galactic latitude. After galactic cut 65% of the sky remains available for analysis. Sum (A+B) and difference (A–B) maps are the sum and difference of the two DMR maps at each frequency. Galactic and cosmic signals cancel out in the (A–B) maps. The mean temperature is subtracted after galactic cut. The resulting temperature standard deviation of the 53 and 90 sum maps are 51 and $69\,\mu$K respectively. Only spots that appear at a threshold $|\nu| \geq 2.3$ were studied.

Hot and cold spots on DMR maps were analyzed by an algorithm that forms tree data structures on binary maps of hot pixels (Torres, 1994e; Torres, 1994c). The statistical significance of a spot is estimated by means of Monte Carlo simulations that take into account instrument noise, sky coverage, pixelization scheme and the

DMR beam characteristics (Torres 1994c). The location of a hot spot is given by the coordinates of the barycenter of the spot. The 'eccentricity' parameter, ϵ, is defined as the ratio of the distances from the center of the spot to the closest and furthest points along its contour. This parameter coincides with the eccentricity of elliptical patterns for large thresholds, but has no straightforward interpretation at low thresholds where contours are highly convoluted.

Shape information for spots smaller or equal than a pixel is lost, and a value of 1.0 is automatically assigned to its area and eccentricity. Areas are given in pixel units, which for DMR are $4\pi/6144 \approx 2.6^\circ \times 2.6^\circ$ per pixel.

Simulated sky maps of the microwave sky were generated using a spherical harmonic expansion for the sky temperature, $\Delta T/T = \sum_l \sum_m a_{lm} W_l Y_{lm}$, with Gaussian random coefficients a_{lm} of zero mean and model dependent variance. The weights W_l for DMR given by Wright *et al.* (1994a) were used. Noise, determined by instrument sensitivity and the number of observations per pixel, is included in the simulations.

With a power spectrum of PDFs $P(k) \propto Q^2 k^n$, the variances of a_{lm} can be determined (Abbot, 1984; Bond and Efstathiu, 1987; Fabbri *et al.*, 1987). Constructing the χ^2 distribution of Monte Carlo simulations derived from an $n = 1$, $Q = 16\,\mu$K it was possible to establish that the signal in the DMR maps is consistent with the hipotesis of being of cosmic origin (Torres, 1994e).

Some of the spots on DMR maps must therefore be cosmic. Even though it is not possible to say whether a spot is real or not, one can assign a statistical significance to each spot using its height information in combination with the number of standard deviations that its area deviates from the expected area of spots on noise maps. The spot mean area and variance in noise maps are found with Monte Carlo simulations.

TABLE I

Hot spot characteristics as appear at different threshold levels (≥ 2.3). The 'significance' S and s_ν parameter are defined in the text. The DMR map is indicated by M. The area, A, is in DMR pixel units. The spot eccentricity is ϵ, and l, b are the galactic longitude and latitude in degrees. The estimated error in pixel location is ±1.5°.

N	S	M	ν	A	s_ν	ϵ	l	b
1	141.5	53	2.33	26	29.1	0.33	4.4	25.5
		53	2.67	16	15.3	0.36	4.8	24.8
		53	3.00	12	8.6	0.26	4.0	24.4
		53	3.33	2	0.1	0.50	2.6	22.5
		53	3.67	1	-0.6	1.00	1.3	22.6
2	45.4	53	2.33	12	11.2	0.60	51.9	65.7
		53	2.67	8	6.1	0.70	54.2	65.6
		53	3.00	2	-0.3	0.50	47.3	65.6
3	15.2	53	2.33	7	4.8	0.37	184.4	-52.3
		53	2.67	4	1.5	0.56	182.6	-52.1
4	14.5	53	2.33	6	3.5	0.74	50.3	38.6
		53	2.67	4	1.5	0.54	51.1	38.6
		90	2.33	4	0.9	0.72	46.4	38.9
5	8.0	53	2.33	1	-2.9	1.00	160.1	-21.3
		53	2.67	1	-2.0	1.00	160.1	-21.3
		53	3.00	1	-1.2	1.00	160.1	-21.3
6	8.0	53	2.33	2	-1.6	0.50	206.8	-20.5
		53	2.67	2	-0.8	0.50	206.8	-20.5
		53	3.00	2	-0.3	0.50	206.8	-20.5
7	5.0	53	2.33	4	1.0	1.00	194.6	44.1
		53	2.67	1	-2.0	1.00	195.6	42.3
8	5.0	53	2.33	2	-1.6	0.50	212.6	-22.1
		53	2.67	1	-2.0	1.00	214.1	-21.9
9	5.0	53	2.33	3	-0.3	0.37	258.2	-22.1
		53	2.67	1	-2.0	1.00	258.2	-22.1
10	5.0	90	2.33	2	-1.6	0.50	7.9	-25.1
		90	2.67	2	-0.8	0.50	7.9	-25.1
11	2.3	53	2.33	1	-2.9	1.00	340.3	51.3
12	2.3	53	2.33	1	-2.9	1.00	208.6	30.7
13	2.3	53	2.33	1	-2.9	1.00	310.5	35.7
14	2.3	53	2.33	1	-2.9	1.00	175.0	-49.6
15	2.3	53	2.33	1	-2.9	1.00	14.8	-48.8
16	2.3	90	2.33	2	-1.6	0.50	248.6	51.0
17	2.3	90	2.33	4	0.9	0.95	164.1	23.1
18	2.3	90	2.33	1	-2.9	1.00	194.9	33.1
19	2.3	90	2.33	1	-2.9	1.00	175.5	-46.5
20	2.3	90	2.33	2	-1.6	0.50	160.7	-60.4

TABLE II

Most significant cold spots. Columns have the same meaning as in Table I.

N	S	M	ν	A	s_ν	ϵ	l	b
1	45.2	53	-3.33	1	-0.8	1.00	242.6	46.4
		53	-3.00	5	2.1	0.64	242.9	45.2
		53	-2.67	8	5.9	0.50	240.7	44.3
		53	-2.33	10	8.5	0.45	241.4	44.8
2	32.6	90	-3.00	4	1.3	0.84	236.5	36.8
		90	-2.67	5	2.1	0.60	237.3	37.3
		90	-2.33	11	9.9	0.64	238.0	36.0
3	23.1	90	-3.00	5	2.1	0.58	126.1	-51.3
		90	-2.67	5	2.1	0.58	126.1	-51.3
		90	-2.33	7	4.8	0.46	126.8	-51.1
4	16.7	53	-2.33	1	-2.8	1.00	256.3	64.0
		90	-2.67	4	1.2	0.97	247.6	70.1
		90	-2.33	7	4.8	0.44	250.3	68.2
5	14.7	53	-2.67	4	1.4	0.51	336.0	-20.9
		53	-2.33	7	4.7	0.44	336.5	-22.1
6	13.6	53	-2.67	1	-1.9	1.00	280.0	55.5
		53	-2.33	7	4.7	0.42	281.2	55.8
7	13.6	53	-2.67	1	-1.9	1.00	275.1	75.5
		53	-2.33	7	4.7	0.52	277.3	74.3
8	5.1	90	-2.33	5	2.2	0.42	100.9	-67.6
9	5.0	53	-2.67	2	-0.8	0.50	302.3	41.8
		53	-2.33	3	-0.3	0.83	302.7	42.7
10	5.0	90	-2.33	4	1.0	0.50	242.4	22.9
		90	-2.67	2	-0.8	0.50	243.2	23.0
11	5.0	90	-2.67	1	-1.7	1.00	254.6	39.2
		90	-2.33	2	-1.6	0.50	253.3	39.0
12	2.3	53	-2.33	2	-1.6	0.50	288.4	66.4
13	2.3	53	-2.33	1	-2.8	1.00	70.0	-26.7
14	2.3	53	-2.33	1	-2.8	1.00	72.8	-24.3
15	2.3	53	-2.33	1	-2.8	1.00	146.3	-41.3
16	2.3	53	-2.33	1	-2.8	1.00	351.0	-52.5
17	2.3	53	-2.33	2	-1.6	0.50	87.0	-46.5
18	2.3	90	-2.33	1	-2.8	1.00	301.4	54.8
19	2.3	90	-2.33	2	-1.6	0.50	270.0	61.8
20	2.3	90	-2.33	1	-2.8	1.00	345.4	-27.4
21	2.3	90	-2.33	1	-2.8	1.00	358.7	28.2
22	2.3	90	-2.33	1	-2.8	1.00	88.6	-40.2
23	2.3	90	-2.33	1	-2.8	1.00	197.3	-27.1
24	2.3	90	-2.33	1	-2.8	1.00	84.0	-55.8

The area, location and eccentricity of the most significant hot and cold spots are in Tables I and II. The spots are ordered according to a 'significance parameter' defined as $S = \sum_{\nu} |\nu| s_{\nu}$, where s_{ν} is the number of standard deviations the spot area deviates from the mean spot area of noise maps at threshold ν. If $s_{\nu} < 1.0$ it is set to 1.0 when computing S. The parameter S is an assessment of the relative probability that a spot may be of cosmic origin.

At the 2.3σ level there are no spots seen simultaneously on the three (A+B) DMR maps. Only one hot spot (No. 4) and one cold spot (No. 4) consistently appear on both 53 and 90 (A+B) maps. Hot spot No. 1 in 53 (A+B) map is clearly associated with the galactic bulge at $l = 0°$ seen in this map. The only features on the 31 GHz sum map that may be significant are a hot spot coincident with hot spot No. 6. and one cold spot coincident with cold spot No. 10. With the exception of hot spot No. 2 and the two spots that appear in two maps (Hot No. 4 and Cold No. 4) at high ν, all other features are marginaly significant. Eliminating hot spot No. 1 (galaxy) and all spots below $\nu = 2.6$ there remains 9 hot spots and 11 cold spots, consistent with the expected results from simulations of cosmic signal ($n = 1$, $16\,\mu$K Quadrupole). With the four year DMR data the signal-to-noise ratio will be greater than one. thus cosmography will become even more relevant, and it will be possible to confirm or discard the identified spots. Bennet *et al.* (1993b) have shown that the features on the first year maps are not correlated with known astronomical sources, thus if the spots are real. their most likely origin is cosmic.

4 The Shape of the Primordial Spectrum According to COBE

Subtracting the dipole component from the CMB maps and excluding the galactic plane, one should see in the CMB temperature distribution the signature of several possible effects of astrophysical and/or cosmic origin (Torres. 1994e). Being the remotest probe we have into the early universe, the CMB data has become an essencial element in testing cosmological models and models of structure formation at large scales. The first analysis of COBE's data gave a strong support for inflationary models ($n = 1$). Mixed dark matter models and models with cosmological constant were not excluded (Wright *et al.*, 1992). However. due to the limited sensitivity of experiments. the intrinsic uncertainty in the determination of some cosmological parameters due to cosmic variance, galactic contamination. and experimental complexities, the task of obtaining useful information for cosmology becomes very difficult.

The fact that the nature of the physical process that generated the CMB anisotropies is a stochastic one (i.e. fluctuations of the gravitational potential on the surface of last scattering) the data that become available to us correspond to just one realization of one universe while theories predict mean values (i.e. $\langle a_l^2 \rangle$). Uncertainties in the two point correlation function and in other higher moments due to cosmic variance have been calculated (Scaramella and Vittorio. 1990; Cayòn *et al.*, 1991; White *et al.*, 1993; Srednicki, 1993; Gutièrrez de la Cruz *et al.*, 1994). Similarly, the cosmic variance uncertainty in the determination of n based on a genus analysis was found to be $\delta n = 0.2$ (Torres, 1994b). In order to deal with this problem it has been suggested to use a more adequate statistic: the number of cosmic observers (Scaramella and Vittorio, 1993), or the average χ^2 (Torres, 1994b) which use 'observable' quantities to compare data with models.

For this reason, the task of obtaining information of relevance for cosomological model testing becomes very difficult, and powerful and sofisticated statistical technques are required (Bond, 1994).

4.1 THE COHERENCE ANGLE OF THE RADIATION FIELD

Using the harmonic coefficients of the expansion of $\Delta T/T$ the CMB correlation function $C(\theta)$ and the angular power spectrum C_l can be written:

$$C(\theta) = \frac{1}{4\pi} \sum_{l=2} (2l+1) C_l P_l(\cos(\theta)), \tag{1}$$

$$C_l = \sum_m |a_{lm}|^2. \tag{2}$$

The coherence angle of the field is $\theta_c = (-C''(0)/C(0))^{-1/2}$ and can be obtained by means of the genus, or total curvature of iso-temperature contours. The genus topological descriptor, G_ν, as a function of threshold level ν depends only on the coherence angle of the field:

$$\langle G_\nu \rangle = \left(\frac{2}{\pi}\right) \frac{\nu}{\theta_c^2} \exp(-\frac{\nu^2}{2}) \tag{3}$$

The genus statistic is an unbias estimator of the coherence angle (Adler, 1981), and the coherence angle is a relevant quantity to characterize a Gaussian random 2D field. In the general case of a sky map including signal and noise θ_c is given by:

$$\theta_c^{-2} = \frac{\theta_{noise}^{-2} + r\theta_{signal}^{-2}}{1+r}, \tag{4}$$

with r the signal-to-noise ratio:

$$r = \frac{\sum_l C_l^{signal}}{\sum_l C_l^{noise}} = \frac{\sigma_{signal}^2}{\sigma_{noise}^2}. \tag{5}$$

The coherence angle of the signal component is directly connected to theoretical models through the C_l coefficients:

$$\theta_{signal}^{-2} = -\frac{1}{2} \frac{\sum_l C_l l(l+1)}{\sum_l C_l}. \tag{6}$$

From the above formula it can be seen that the coherence angle is a quantity less affected by cosmic variance: because of the factor $l(l+1)$ weighting the C_l, the contribution of the low l terms (those with the largest cosmic variance) is reduced relative to the high l terms.

4.2 TESTING COSMOLOGICAL MODELS WITH CMB DATA

Since CMB large angle scale maps probe the spectrum of PDFs they provide a direct way to test models that predict a particular value of the spectral index n. The amplitude of the spectrum of PDFs Q is not independent of n due to the degeneracy caused by the limited range of l space available to COBE and the presence of noise and cosmic variance. However if one can fix Q based on an independent determination, then one can investigate the range of n consistent with the data. Inflation predicts a scale invariant spectrum i.e. $n = 1$ (from scalar perturbations), however if the contribution of tensor fluctuations is not negligible then $n < 1$ (Crittenden *et al.*, 1993). Tilted spectra are also predicted in other flavors of inflation (Cen *et al.*, 1992) and in mixed darke matter models (Pogosyan and Starobinsky, 1994). The importance of an accurate determination of n is well established.

The value of n which best fits the two year DMR 53(A+B) map was obtained based on the genus characteristic of these maps. The method consists of calculating the probability $P_{53} \equiv P(\chi^2 > \chi^2_{53})$ of obtaining a χ^2 statistic larger than the same statistic derived from the DMR two year 53(A+B) map, χ^2_{53}, for different values of n and keeping the amplitude Q equal to $12\,\mu$K as in Bennett *et al.* (1994). For each model (defined by n) the genus parameter was computed for 400 Monte Carlo realizations of COBE second year sky maps (i.e. maps prepared following the same procedure as for the hot/cold spots analysis but including corresponding noise for two year maps). For each realization the χ^2 statistic was obtained:

$$(\chi^2)^k = \sum_{i=1}^{25}\sum_{j=1}^{25}(G_i - \langle G_i\rangle)M_{ij}^{-1}(G_j^k - \langle G_j^k\rangle), \qquad (7)$$

with G_i^k the genus of the k-th realization at threshold level i. A total of 25 equaly spaced threshold levels between -3.0 and 3.0 were used. The covariance matrix is also formed with the genus of the N Monte Carlo sky maps:

$$M_{ij} = \frac{1}{N-1}\sum_{k=1}^{N}(G_i^k - \langle G_i\rangle)(G_j^k - \langle G_j\rangle) \qquad (8)$$

with $\langle G_i\rangle$ the mean value of the genus at the i-th thershold bin.

From Table III. with the values of P_{53} for different n it can be seen that the best fit model is $n = 1.2$ with a probability $P_{53} = 77.8\%$, but other values of n also result in a good fit to the data: the interval $n = 0.8 - 1.3$ corresponds to probabilities (P_{53}) greater or equal to 68%. This result is consistent with previous estimates based on different statistical techniques (see table IV).

It is clear from Table IV that the results so far obtained for n are not stringent enough to be able to select a particular flavor of inflation models or mixed dark matter models with tilted spectrum. To narrow the interval in n consistent with experiment it will be necessary to incorporate into the analysis data on medium and short scale matter clustering derived from galaxy catalogs.

TABLE III

Probability of obtaining $\chi^2 > \chi^2_{53}$, P_{53} under different hypothesis parametrized by n with fixed amplitude $Q = 12\,\mu$K.

n	χ^2_{53}	P_{53}
0.200	22.622	55.379
0.400	22.356	60.722
0.600	20.437	72.428
0.800	20.801	67.698
1.000	20.170	74.321
1.200	19.413	77.784
1.400	20.322	70.296
1.600	22.074	62.682
1.800	21.101	65.805
2.000	25.787	43.731
2.200	24.944	45.220
2.400	26.080	38.020
2.600	25.592	45.283
2.800	28.047	29.969

TABLE IV

Primordial spectrum amplitude Q in μK and slope n derived from COBE data using different analysis techniques. $Q(n)$ explicitely represents the $Q-n$ degeneracy. References are: 1 - Smoot *et al.* 1992; 2 - Bennett *et al.* 1994; 3 - Smoot *et al.* 1994; 4 - Wright *et al.* 1994b; 5 - Gòrski *et al.* 1994; 6 - Torres *et al.* 1994d; 7 - Hu *et al.* 1994; 8 - Wright *et al.* 1994a; 9 - Torres 1994e; 10 - Bond 1994; 11 - Torres *et al.* 1994b; 12 - Seljak 1993.

Method	Q	n	$Q(n)$	reference
Fit to $C(\theta)$	17 ± 5	1.1 ± 0.5		1
Max. likelihood, $C(\theta)$	$12.4^{+5.2}_{-3.3}$	$1.59^{+0.49}_{-0.55}$		2
Fit to σ_{sky}	13.2 ± 2.5	$1.7^{+0.3}_{-0.6}$		3
Power 53 × 90		1.46 ± 0.4		4
Power 53		1.24 ± 0.36		5
Power 90		1.29 ± 0.46		5
Fit to $C(\theta)$		0.76 ± 0.3		6
Fit to σ_{sky}		< 1.6		7
Fit to σ_{sky}	17.1 ± 2.9	1 fixed		8
Genus statistic	16 fixed	1.2 ± 0.3		9
S/N eigen-mode		1.8 ± 0.5		10
Genus statistic			$15.7 \pm 2.2 - (16.6 \pm 0.3)(n-1)$	3
Genus statistic			$22.2 \pm 1.7 - (4.7 \pm 1.3)n$	11
Max. likelihood, $C(\theta)$			$15.7 \exp(0.46(1-n))$	12

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