The power spectrum implied by COBE and the matter correlation function

S. Torres[1]*, **R. Fabbri**[2], and **R. Ruffini**[1]

[1] ICRA (International Center for Relativistic Astrophysics), Dipartimento di Fisica, Università di Roma 1, Pzza. Aldo Moro 2, I-00185 Roma, Italy, e-mail: 40174::TORRES

[2] Sezione di Fisica Superiore, Dipartimento di Fisica dell'Università di Firenze, Via S. Marta 3, I-50139 Firenze, Italy

Abstract. A phenomenological power spectrum of primordial density perturbations has been constructed by using both COBE data to probe the large wavelength region, and a double power law, locally deduced from galaxy catalogs, which describes the matter correlation function up to tens of Megaparsec. The shape of the spectrum $P(k)$ of density fluctuations exhibits a peak that singles out a characteristic wavelength λ_{peak} proportional to the cutoff radius R_0 in the matter distribution (comparable to the distance at which matter becomes anticorrelated). From a least squares fit to COBE's angular correlation function we obtain $R_0 = 35 \pm 12h^{-1}$Mpc for the correlation length, and $n = 0.76 \pm 0.3$ for the spectral index of $P(k)$ in the large wavelength region. The inferred scale in the spectrum is $\lambda_{peak} = 51 \pm 18h^{-1}$Mpc. This number agrees with that derived from the analysis of the correlation function of matter and with a preferred scale identified in IRAS PSC.

Key words: galaxies: clustering – cosmic microwave background – large-scale structure of universe – galaxies: formation

1. Introduction

After the positive detection of anisotropies in the cosmic microwave background radiation (CMB) by COBE-DMR (Bennett et al. 1992; Smoot et al. 1992), it is now possible to directly probe the spectrum of primordial density fluctuations (PDF). Early attempts to place restrictions on the spectrum of PDF relied mainly on the existing data on large scale matter distribution or on theoretical prejudices, depending on what region of the spectrum was explored. While matter distribution is useful in determining the shape of the 'evolved' spectrum at relatively large wave-numbers, large angular scale CMB anisotropies are sensitive to the primordial spectrum at low wave-numbers.

Analysis of the QDOT IRAS surveys and the angular two-point autocorrelation function $w(\theta)$ (Peacock 1991) shows that both data sets are consistent with a spectral function $P(k)$ that approaches $k^{-1.4}$ for large k. However, increasing the depth of the surveys has revealed the presence of a break in $\xi(r)$ (the two-point galaxy-galaxy correlation function; Calzetti et al. 1992), the existence of a region where $\xi(r)$ is negative (Guzzo et al. 1991), and also the possible detection of a cutoff radius R_0 of order 30 h^{-1} Mpc. In this paper we make use of the double power law in $1 + \xi(r)$ to derive a properly normalized power spectrum for large k, and we match it to a power law $P(k) \propto k^n$ for small k. The matching point is selected so that the power spectrum makes a smooth transition from the primordial power law to the 'evolved' spectrum. As a result, the peak in the spectrum turns out to be of the same order of magnitude as R_0. The cutoff radius in the matter distribution R_0 and the spectral index n are then the only two important quantities, and we can treat them as free parameters. A calculation of anisotropies in the CMB with this model for the power spectrum allows us to fit these parameters to COBE's angular correlation function. The quadrupole amplitude allows for an independent check for consistency.

The fitting turns out to be very satisfactory and provides for R_0 a best value of $35 \pm 12h^{-1}$Mpc very close to the optimal value inferred from observations of the galaxy-galaxy correlation function (Calzetti et al. 1991). The best value for the primordial spectral index is found to be 0.76 ± 0.3, with a central value somewhat lower than predicted by standard exponential inflation but, because our error bars, still consistent with it and with Smoot et al. (1992). A relatively low index is not peculiar to the model because for a pure power-law spectrum we find similar results. With respect to this, we notice that an attempt to fit COBE data to a power law spectrum of density fluctuations within CDM models results in a spectral index $0.6 < n < 1.0$ (Adams et al. 1992). Kashlinsky (1992) also finds $n < 1.0$ in order to satisfy data on both the large scale galaxy distribution and large scale streaming motions.

A feature which, on the other hand, is peculiar to our model is the close relationship between the cutoff radius of the matter

---

correlation function and the peak in the spectrum. As a matter of fact, the maximum of $P(k)$ is located at $\lambda_{\text{peak}} = 51 \pm 18 h^{-1}\text{Mpc}$. This number agrees with the privileged length scale of about $50h^{-1}\text{Mpc}$ found in the IRAS Point Source Catalog by Fabbri & Natale (1993a). A further analysis (Fabbri & Natale 1993b) provides evidence for the interpretation of this scale as a peak in the product $k^{1/2}P(k)$. However, it does not agree with the very large scales recently inferred from redshift surveys (Fisher et al. 1993; Einasto et al. 1993; Jing & Valdarnini 1993). This point will be discussed in Sect. 5.

2. Power spectrum

To construct a phenomenological spectrum we started from the autocorrelation of the density fluctuation $\delta\rho/\rho$, which is equal (up to a delta function at the origin) to the particle correlation function $\xi(r)$ and can be parametrized as follows (Calzetti et al. 1992):

$$1 + \xi(x) = \begin{cases} A_1 x^{-\gamma_1} & x < x_t \\ A_2 x^{-\gamma_2} & x_t < x \le x_0 \\ 1 & x > x_0 \end{cases} \qquad (1)$$

To make the calculations independent of Hubble's constant $H_0 = 100h \text{ Km s}^{-1} \text{ Mpc}^{-1}$ we adopt the change of variable $x = hr$. The correlation function admits four independent parameters, which we choose to be the spectral indexes γ_1 and γ_2 and the transition points x_t and x_0. The amplitudes A_1 and A_2 are then calculated in terms of these parameters by demanding continuity of $1 + \xi(x)$ at $x = x_t$ and the normalization condition on the correlation function $\int_V \xi(r)r^2 dr = 0$. The resulting formulae for the amplitudes are:

$$A_2 = \frac{3 - \gamma_2}{3} x_0^{\gamma_2} \left[1 + \frac{\gamma_1 - \gamma_2}{3 - \gamma_1} \left(\frac{x_t}{x_0} \right)^{3 - \gamma_2} \right]^{-1} \qquad (2)$$

$$A_1 = A_2 x_t^{(\gamma_1 - \gamma_2)}. \qquad (3)$$

Three parameters are determined by the existing data, namely $\gamma_1 = 1.8$, $\gamma_2 = 0.8$, $x_t = 3.0$ Mpc. It turns out that our results are quite insensitive to the precise values of γ_1 and x_t, and moderate differences are found varying γ_2 in the range $0.7 - 0.9$. On the other hand, the value of the cutoff radius $x_0 = hR_0$ is important for our calculations because the dependency of A_1 and A_2 on this parameter affects the normalization of $P(k)$ and thereby the CMB anisotropies. We treat x_0 as a free parameter since it is still poorly determined.

Let us now define the spectrum of PDF in terms of the rms density fluctuations:

$$\left(\frac{\delta\rho}{\rho} \right)^2 = \int P(k)\Phi_k^2(\eta)\mathrm{d}^3 k, \qquad (4)$$

where k is a dimensionless wavenumber defined as:

$$k = \frac{4\pi c}{H_0 \lambda} = \frac{k}{h\alpha}, \qquad (5)$$

with $\underline{k}$ the physical comoving wavenumber, $\alpha \equiv 1/6000 \text{ Mpc}^{-1}$, and η the conformal time. The temporal evolution of perturbations is described by $\Phi_k(\eta)$, which in the linear regime is independent of wavenumber, and for $\Omega = 1$ can be set equal to $\eta^2/10$ with $\eta_o = 1$. The spectrum of PDFs, being the Fourier transform of the matter correlation function (Peebles 1980), results in

$$P(k) = \frac{\alpha^3}{2\pi^2 \Phi^2(\eta_o)} \int_0^{x_0} x^2 \xi(x) \frac{\sin(\alpha kx)}{\alpha kx} \mathrm{d}x. \qquad (6)$$

The above integration was done numerically using a combination of both the Clenshaw-Curtis and Gauss-Kronrod methods which are adequate for an oscillating integrand (Piessens et al. 1983). The results of the Fourier transform shows the presence of a peak in $P(k)$ at a wavenumber k_{peak} inversely proportional to x_0^{-1} ($k_{\text{peak}} \approx 2.56 \times 10^4 \text{Mpc}/x_0$). Beyond the peak the function $P(k)$ shows an oscillatory behavior (due to the sharp edge cutoff of $\xi(r)$) at first, but eventually reaches a power law behavior with a negative slope $P(k) \propto k^{-1.2}$, in agreement with the results of Peacock (1991). For small wavenumbers $k < k_{\text{peak}}$ we find $P(k) \propto k^2$. This asymptotic law would be found from any correlation function which is forced to vanish beyond a maximum radius x_0, or decreases as $x^{-\gamma}$ with $\gamma > 5$, provided the integral of ξ over space vanishes. This behavior does not agree with COBE's data. However, since the domain where the galaxy-galaxy correlation function appreciably differs from zero is limited to relatively small scales, $P(k)$ can be modified in the small-k region without appreciably affecting $\xi(x)$. Therefore we replaced the $P(k) \propto k^2$ law by a power function with a spectral index n left as a free parameter to fit with COBE's data. In order to minimize the perturbation to $\xi(x)$ we extrapolated the spectrum in Eq. (6) down to the wavenumber k_m where its slope assumes the desired value of n. For $k < k_m$ the power law function $P(k) = P_m(k/k_m)^n$ was adopted. The parameter P_m is the value of the original, ξ-derived spectrum in $k = k_m$. Continuity of both the spectral function and its first derivative is therefore guaranteed. Figure 1 illustrates the form of the function $P(k)$ used here for our best values of x_0, n and for $\gamma_2 = 0.8$. We have found a power series solution to the Fourier transform in Eq. (6) neglecting the region where $\xi(r)$ has slope -1.8:

$$P(k) = \frac{200}{3\pi^2}(\alpha x_0)^5 \gamma_2 k^2 \\ \times \sum_{i=1}^{\infty} (-1)^{i+1} \frac{i(i+1)}{2i+3-\gamma_2} \frac{(\alpha x_0 k)^{2i-2}}{(2i+3)!}, \qquad (7)$$

which can be conveniently used to find the matching point k_m of the function an its first derivative. An approximation which is sufficient to our purpose is

$$k_m^2 = \frac{14}{(\alpha x_0)^2} \frac{2-n}{4-n} \frac{7-\gamma_2}{5-\gamma_2} \\ \times \frac{1 - \frac{7}{18}F(n)G(\gamma_2)}{1 - \frac{7}{9}F(n)G(\gamma_2)} \qquad (8)$$
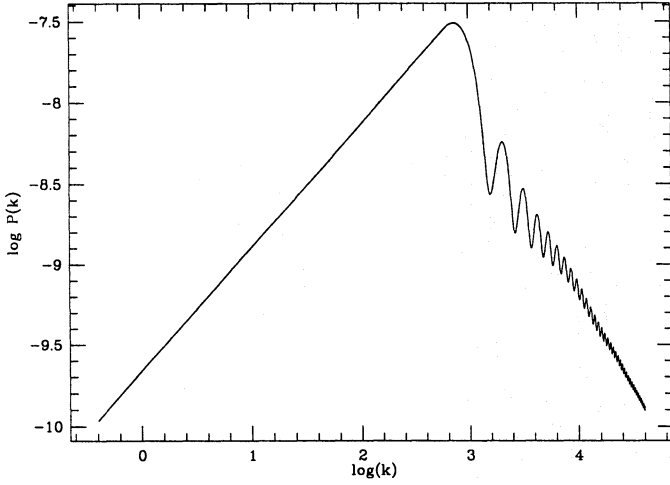
$$F(n) = \frac{(2-n)(6-n)}{(4-n)^2} \qquad (9)$$

Fig. 1. Spectrum of primordial density fluctuations for $x_0 = 34.8$ Mpc, $n = 0.76$ and $\gamma_2 = 0.8$

$$G(\gamma) = \frac{(7 - \gamma)^2}{(5 - \gamma)(9 - \gamma)}. \tag{10}$$

Since Eqs. (6)-(8) do not contain x_t and γ_1, they cannot describe the large-k region. However, this region is quite ineffective to influence the CMB anisotropies.

3. CMB anisotropies

With a properly normalized spectrum of PDF, one can calculate the CMB large scale fluctuations and angular correlation function. The anisotropy field is conveniently expressed in terms of the coefficients $a_{\ell m}$ in a harmonic expansion:

$$\frac{\Delta T}{T_0} = \sum_{\ell \geq 2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi), \tag{11}$$

with the dipole terms ($\ell = 1$) excluded. For random perturbation fields the variance of $a_{\ell m}$ defines the angular power spectrum:

$$\langle a_\ell^2 \rangle = \sum_{m=-\ell}^{\ell} \langle a_{\ell m}^2 \rangle$$

$$= (2\ell + 1)\langle a_{\ell m}^2 \rangle. \tag{12}$$

Experiments such as COBE-DMR which produce a sky map of the cosmic background radiation give a direct measure of the angular correlation function $C(\theta, \sigma)$, which can be theoretically predicted by

$$C(\theta, \sigma) = \frac{1}{4\pi} \sum_\ell \langle a_\ell^2 \rangle W_\ell^2 P_\ell(\cos \theta), \tag{13}$$

where $P_\ell(\cos \theta)$ are the Legendre polynomials, and the 'filter' function for a Gaussian beamwidth $\sigma = 1/(\ell_0 + \frac{1}{2})$ radians is

$$W_\ell = \exp\left[-\frac{(\ell + \frac{1}{2})^2}{2(\ell_0 + \frac{1}{2})^2} \right]. \tag{14}$$

Table 1. Best fitted parameters

γ_2	x_0 (Mpc)	n	χ^2
0.7	36.4 ± 12	0.76 ± 0.3	64.055
0.8	34.8 ± 12	0.76 ± 0.3	64.055
0.9	33.5 ± 12	0.76 ± 0.3	64.055

For COBE-DMR filtering effectively starts at $\ell_0 \approx 18$ (corresponding to an angular resolution of $3.1°$), but in our case terms up to $\ell \approx 60$ should be included for an accurate calculation.

The imprint left on the CMB produced by metric perturbations during the epoch of last scattering, (Sachs & Wolfe 1967) can be calculated by evaluating the integral:

$$a_\ell^2 = \left(\frac{4\pi}{5} \right)^2 (2\ell + 1) \int P(k) \left[\frac{j_\ell(k)}{k} \right]^2 dk, \tag{15}$$

with j_ℓ the spherical Bessel functions. For a cutoff radius x_0 lying in the range from 20 to 100 Mpc the location of the peak of $P(k)$ goes from $k_{\text{peak}} = 1.3 \times 10^3$ to 256. In our calculations the shape of $P(k)$ around the maximum is not so important for the determination of the anisotropies of the CMB, other than providing an overall normalization.

4. Results

The possibility of using the DMR angular correlation data to find restrictions on the spectrum of PDF was exploited here by fitting the theoretical angular correlation function $C(\theta, 3.1°; x_0, n)$ *without any arbitrary normalization factors* to that obtained by COBE-DMR. For each value of γ_2 the parameters resulting from the fit are the cutoff radius x_0 and the spectral index n of the long-wave side of PDF spectrum.

A numerical minimization of the χ^2 function using CERN's MINUIT package (James & Roos 1975) gives the results reported in Table 1.

The fitted parameters are found to be fairly insensitive to the precise value of γ_2, so that we can safely assume that $x_0 = 35 \pm 12$ Mpc and $n = 0.76 \pm 0.3$. The errors quoted correspond to the 68% confidence level. The χ^2 reported are for $69 - 2$ degrees of freedom. Figures 1, 2 and 3 respectively show the power of density fluctuations, the CMB angular power spectrum, and the correlation function obtained by the fit for the $\gamma_2 = 0.8$ case. In Fig. 3 the DMR data is also included in the plot. Figures 4 and 5 give contour plots of χ^2 near the minimum. The contours show that the two-parameter fit exhibits a strong correlation. For $\gamma_2 = 0.8$ we have $x_0 = 0.48 + 45.54n$ Mpc, with $r = 0.9996$. This effect is built-in by the way we normalize the galaxy-galaxy correlation function. From Eqs. (2) and (3) it appears that increasing x_0 the amplitude of the galaxy-galaxy correlation function and thereby of $P(k)$ increases, so that we need to increase n in order to suppress the excess of long-wave power.
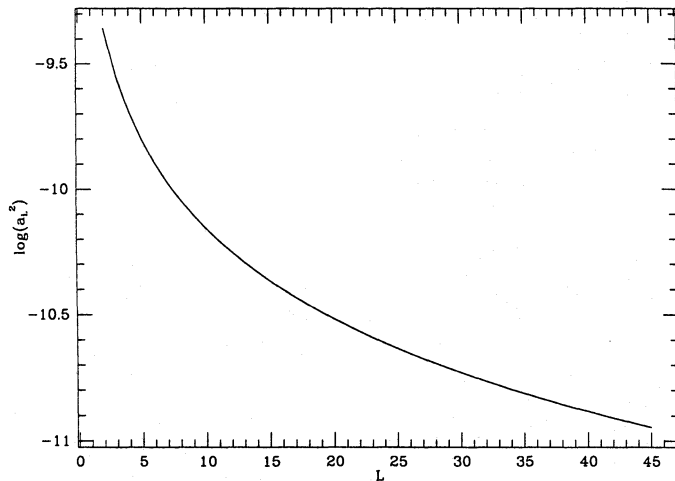
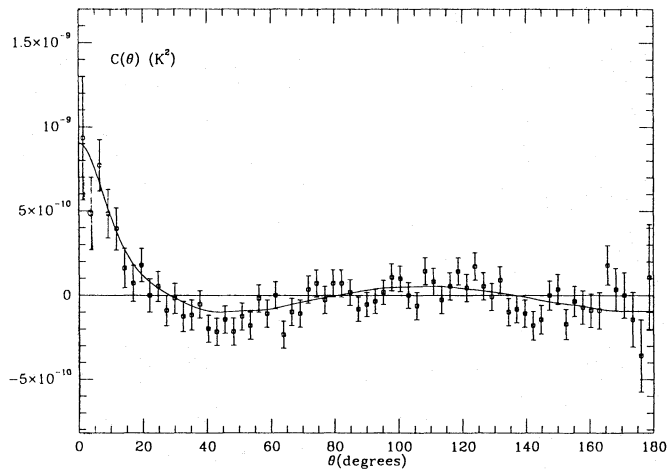**Fig. 2.** Angular power spectrum for the best fit values as in Fig. 1

Fig. 3. Angular correlation function for best fit values as in Fig. 1 and COBE-DMR data points. The dipole and quadrupole terms have been removed
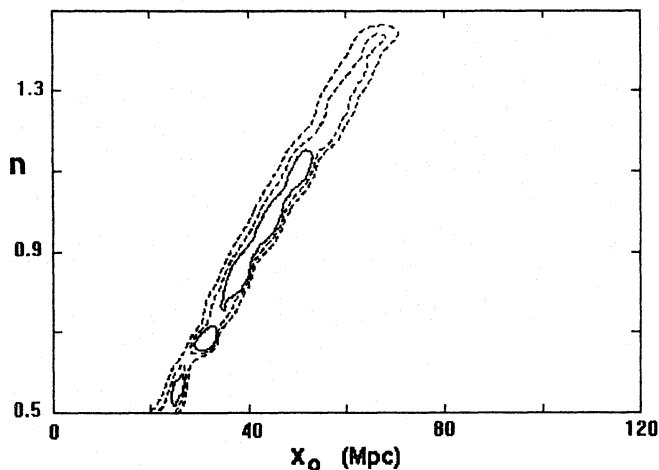

Fig. 4. Contour plot of χ^2 around the minimum for $\gamma_2 = 0.8$. The levels are for $\Delta\chi^2 = 2.3, 6.17$ and 11.8, corresponding to 68.3%, 95.4% and 99.7%
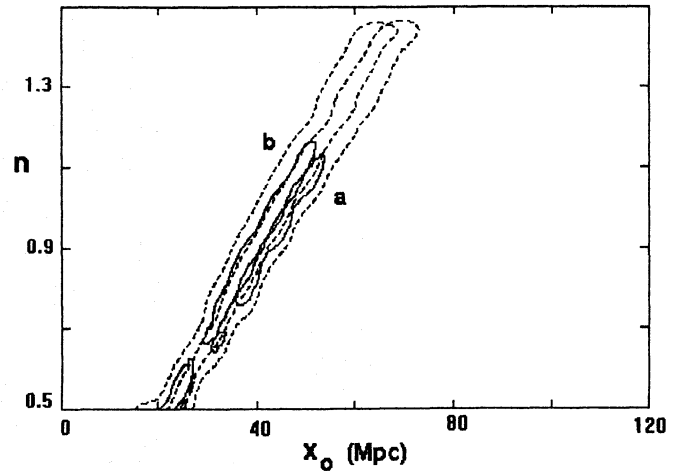

Fig. 5. Contour plots for a) $\gamma_2 = 0.7$, and b) 0.9. The reported contours are for $\Delta\chi^2 = 2.3$ and 11.8

One could argue that if the cutoff radius x_0 is an independently well known parameter, then it should be fixed. Taking $x_0 = 30$ Mpc (the best value of Calzetti et al. 1992) the spectral index obtained from the fit is more tightly constrained: $n = 0.64 \pm 0.02$ with a χ^2 of 64.2. This is far from unity.

Alternatively one could fix n to 1.0 and ask for the best x_0 that fits the data, still satisfying the second of the conditions above. This results in $x_0 = 44 \pm 1h^{-1}$ Mpc with $\chi^2 = 64.88$. This radius implies $\lambda_{peak} = 65h^{-1}$ Mpc.

To compare with the numbers obtained by the COBE-DMR group we have made the fit using a 'pure', unnormalized power law $P(k) \propto k^n$, and left the quadrupole amplitude a_2^2 as a free parameter of the fit (which in the end is used to renormalize the spectrum). The results are $\chi^2 = 64.06$, quadrupole amplitude, $Q_{rms-ps} = (\Delta T)_2 = T_0(a_2^2/4\pi)^{1/2}$, of 16 ± 8 μK and $n = 0.8 \pm 0.3$. If one fixes n to 1.0 then χ^2 increases to 64.88 while the quadrupole amplitude decreases to $Q = 15 \pm 5$ μK.

We have checked the consistency of our results with the independent measurement of the RMS quadrupole anisotropy by COBE-DMR. As can be seen in Fig. 6, the quadrupole evaluated at the optimal values of x_0 and n agrees with COBE's Q_{rms-ps}.

5. Discussion

The cutoff radius $R_0 = 35 \pm 12h^{-1}$ Mpc agrees very well with the result found by Calzetti et al. (1992) on the Zwicky/CfA1, SSRS, and IRAS PSC catalogs. Since R_0 corresponds to a wavelength of $51h^{-1}$ Mpc it agrees with the preferred wavelength found by Fabbri & Natale (1993a) in IRAS PSC, $\lambda = 50h^{-1}$ Mpc. However, the best value for the primordial spectral index n is found to be somewhat lower than the best value declared by Smoot et al. (1992), and from that expected from the scenario of standard inflation. The possibility of lower values of n from DMR data has been noticed also by Adams et al. (1992) and Kashlinsky (1992).

Is our analysis consistent with standard inflation? Inspection of our χ^2 contours in Figs. 4 and 5 shows that it is so only paying
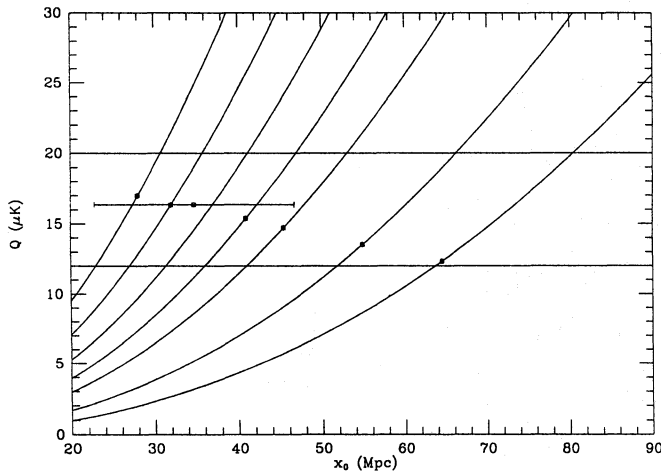
Fig. 6. Quadrupole amplitudes derived from the model with $\gamma_2 = 0.8$ for various values of n. Solid lines are curves of fixed n, with $n = 0.6$, 0.7, 0.8, 0.9, 1.0, 1.2 and 1.4 going from the steepest to the least steep curve. The quadrupole amplitudes corresponding to the optimal value of x_0 are indicated by squares. The error bars at the absolute minimum correspond to the 68% confidence level. The horizontal lines indicate the 68% confidence region of COBE's Q_{rms-ps}

the price of a rather large value of the cutoff radius, $R_0 \approx 45h^{-1}$ Mpc. Such a value weakens the agreement with Calzetti et al. (1992). However we should notice that our model - obtained by the requirement of a minimum perturbation of $\xi(r)$ - forces the test to be more stringent than other tests on DMR data since there is no normalization factor as a free parameter. We cannot exclude that different spectral shapes might provide results more favourable towards standard inflation, also a pure power-law model with a best-fitted normalization factor does favor values of n smaller than unity. It is worth noticing that so small values of n also arise theoretically from power-law inflation (Fabbri et al. 1987).

While the best value found for the correlation scale is completely satisfactory, the peak wavelength found for $P(k)$ is not consistent with spectra recently derived from redshift surveys on IRAS galaxies (Fisher et al. 1993), clusters (Einasto et al. 1993) and both (Jing & Valdarnini 1993). There the power spectra appear to increase towards long wavelenths, and a turnover appears only at the boundaries of the samples $\lambda \sim 150$ or $180h^{-1}$ Mpc. In our view, the significance of such results is weakened by the criticism of Bahcall et al. (1993), who show that very large distortions must be generated by large-scale peculiar velocities. The ratio of the values of the power spectrum at $\lambda \geq 100h^{-1}$ and $\lambda \leq 10h^{-1}$ Mpc, when calculated in redshift space, may be ~ 10 times higher than the 'true' ratio in real space ! Unfortunately the recovery of the genuine, real-space $P(k)$ is model-dependent there. The role of peculiar velocities is not significant in our work, as well as in Fabbri & Natale (1993a, b). Further, it is not clear whether spectra peaked around $150h^{-1}$ Mpc can be made consistent with COBE's data as well as other anisotropy data available around 1°: Kashlinsky's (1992) results seem to imply that compatibility cannot be achieved for so broad spec-

tra, while the goal could be reached for sharp but unphysical peaks. The present paper shows that the peak wavelength must be much smaller if we wish (i) to make a really satisfactory fitting of COBE's data and (ii) to minimally perturb the spectrum directly originated by $\xi(r)$ in the region where it can be measured.

References

Adams, F.C., et al. 1992, Phys. Rev. D, 47, 426
Bennett, C., et al. 1992, ApJ, 396, L7
Bahcall, N. A., Cen, R. & Gramann, M. 1993, ApJ, 408, L77
Bond, J. R. & Efstathiou, J. 1987, MNRAS, 226, 655
Calzetti, D., Giavalisco, M., Ruffini, R., Taraglio, S., & Bahcall, N. A. 1991, A&A, 245, 1
Calzetti, D., Giavalisco, M., & Meiksin, A. 1992, ApJ, 398, 429
Einasto, J., Gramann, M., Saar, E., & Tago, E. 1993, MNRAS, 260, 705
Fabbri, R., Lucchin, F. & Matarrese, S. 1987, ApJ, 315, 1
Fabbri, R. & Natale, V. 1993a, A&A, 267, L15
Fabbri, R. & Natale, V. 1993b, preprint
Fabbri, R. & Ruffini, R. 1992, A&A, 254, 7
Fisher, K.B., Davis, M., Strauss, M.A., Yahil, A., & Huchra, J.P. 1993, ApJ, 402, 42
Gould, A. 1993, ApJ, 403, L51
Guzzo, L. et al. 1991, ApJ, 382, L5
James, F. & Roos, M. 1975, Computer Phys. Comm., 10, 343
Jing, Y.P. & Valdarnini, R. 1993, ApJ, 406, 6
Kashlinsky, A. 1992, ApJ, 399, L1
Peacock, J. A. 1991, MNRAS, 253, 1p
Peebles, P. J. E. 1980, in: The Large-Scale Structure of the Universe, Princeton University Press, Princeton, NJ
Piessens, R., deDoncker-Kapenga, C. W. Uberhuber & D. K. Kahaner. 1993, QUADPACK, Springer-Verlag, New York
Sachs, R. K. & Wolfe, A. M. 1967, ApJ, 147, 73
Smoot, G. et al. 1992, ApJ, 396, L1
Wright, E. et al. 1992, ApJ, 396, L13